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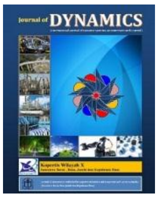

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The Development and Verification of Two Time Domain Identification Algorithms Using Scientific Python Ecosystem – The Free Vibration Problems

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Abstract

This research presented a study for verification the performance and the effectivity of identification program for modal parameters identification. The identification program was written in scientific python ecosystem by implementing Ibrahim Time Domain (ITD) and Eigensystem Realization Algorithm (ERA) method. A cantilever beam with three additional lumped masses was used as the structure under test (SUT). The SUT was modeled to produce numerically simulated responses and tested to obtain experimentally recorded responses. Especially in the first stage, the numerically simulated response was corrupted with white noise with a signal to noise ratio of 12.04 dB to imitate real vibration responses. Then, the performance was marked by capabilities to identify consecutive stable modes for every increment of model order in both stages. Both time domain method presented well those capabilities in the identification process. Hence, it can be concluded that the program can identify well even in high noise contamination in the responses and real vibration data using laboratory test scale.

Keywords: Structural identifications, ambient excitations, ITD method, ERA method, and scientific python ecosystem.

1. Introduction

A structural identification technique using response (output) only or known as operational modal analysis (OMA) is a method which popular this late two decades. The method is easy to apply because no excitation required to excite the structure in artificial means. The structural excitation is obtained in a natural way during its operating or ambient condition. Therefore, this technique gives more advantages for modal parameter identification especially for large-scale structures, such as ships [1], buildings [2], bridge [3], wind turbines [4], [5], rotating machinery [6] and can be applied for structural health monitoring [7].

Those cases are very difficult to conduct if *experimental modal analysis* (EMA) is applied to large-scale structures. EMA needs both input and output to be sampled for performing modal parameter identification. Indeed, input or excitation for such structures are an artificial means with higher load specifications. It makes EMA has a drawback in the term of cost during performing a vibration test. Hence, OMA can be used as an alternative method to modal parameter identifications besides EMA [8].

In general, most of the identification methods used in OMA is the time domain methods. It means that there is no necessary to transform time domain responses into frequency domain responses. The popular time domain identification methods for OMA are Ibrahim time domain (ITD) method [9-12], Eigensystem Realization Algorithm (ERA) method [13] and covariance-driven stochastic subspace identification (SSI-Cov) method [14].

This research presented the study for evaluation the performance of developed time domain identification algorithms using scientific python ecosystem (SPE). The time domain identification algorithm applied in this research were ITD method and ERA method for obtaining structural modal parameters under ambient excitation regarding free vibration responses.

Furthermore, this research was an extension of previous research conducted by Adriyan [15] and Bur, et. al. [16], [17]. Previous researchers had succeeded applying ITD method and random decrement (RD) method along with both combination to identify structural modal parameters using free and forced vibration responses. Identification program based on both methods was developed in MATLAB environment. This identification program had been verified through a study in numerical simulation and laboratory scale test.

Research presented in this paper used same vibration data as the previous research did in [15]. It is aimed to gain a close result between this research and the previous research, especially in term of performance and robustness of developed identification program in SPE. A cantilever beam with three additional masses has used a structure under test (SUT) throughout of this research for identification in numerical simulation or lab scale vibration test. In the numerical simulation, a Gaussian white noise with a signal to noise ratio (SNR) of 12.04 dB was added to the numerical responses. It can be used for simulating real vibration responses which commonly contained noise at a certain degree.

Meanwhile, both algorithms for structural identification were developed in SPE because it is powered by python programming language. Python is an elegant dynamic programming language, easy to write and read, object oriented, can run in all operating system, an open-source software. It is due to nature of python itself, there are a lot of applications written in python for the scientific domain [18]. These scientific applications or modules can be applied to process a computation of numerical data or in symbolic form. Parallel computation in multi-core machine has been supported for numerical calculation. The last is very useful while dealing with the computation involving linear algebra and big data.

2. Material and Methods

In general, identification methods using structural responses can be initiated by representing the dynamic formulation of a structure which possesses a linear time invariant (LTI) characteristics in a continuous time

$$\mathbf{M}\ddot{\mathbf{w}} + \mathbf{D}\dot{\mathbf{w}} + \mathbf{K}\mathbf{w} = \mathbf{f}. \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$, $\mathbf{D} \in \mathbb{R}^{n\times n}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ respectively denote a matrix of structural mass, damping, and stiffness of a system with n degree of freedom and directly correlated with the structural modes. $\ddot{\mathbf{w}} \in \mathbb{R}^n$, $\dot{\mathbf{w}} \in \mathbb{R}^n$, $\mathbf{w} \in \mathbb{R}^n$ and $\mathbf{f} \in \mathbb{R}^n$ described a vector of acceleration, velocity, displacement and external excitation, respectively.

The state space form of eq. (1) can be written as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}\\ \mathbf{y} &= \mathbf{C}\mathbf{x},\end{aligned} \tag{2}$$

where, $\mathbf{A} \in \mathbb{R}^{2n\times 2n}$ and $\mathbf{B} \in \mathbb{R}^{2n\times n}$ are a system matrix and influence input-output matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix},$$

$$\mathbf{B} = [-\mathbf{M}^{-1} \quad \mathbf{0}]^{\mathbf{T}}, \mathbf{x} = (\mathbf{w} \quad \dot{\mathbf{w}})^{\mathbf{T}},$$

and $\mathbf{C} \in \mathbb{R}^{m\times 2n}$ denotes system output matrix which related state responses $\mathbf{x} \in \mathbb{R}^{2n}$ with sensor position, or spatial information of the sampled responses, and the type of sampled responses or also known as observation $\mathbf{y} \in \mathbb{R}^m$.

The use of data acquisition to capture the structural responses puts continuous time state space have to be represented in discrete (digital) time. The application of zero order holds into eq. (2) yields

$$\begin{aligned}\dot{\mathbf{x}}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k\\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k,\end{aligned} \tag{3}$$

where subscript k denotes a time index during the sampling process.

For free vibration problems, right hand side of eq. (1) is equal to zero and makes $\mathbf{u}_k$ has to be zero. Structural responses ($\ddot{\mathbf{w}}$, $\dot{\mathbf{w}}$ or $\mathbf{w}$) in discrete form are stacked to form a block response matrix in the initial phase of the identification process. If $\mathbf{Y}$ is the block response matrix which constructed by the sampled acceleration response

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_- \\ \mathbf{Y}_+ \end{bmatrix}, \tag{4}$$

where $\mathbf{Y}_-$ and $\mathbf{Y}_+$ respectively denote the past and the future of the block response matrix. Both of block response matrix, $\mathbf{Y}_-$ and $\mathbf{Y}_+$, are described as following

$$\mathbf{Y}_- = \begin{bmatrix} \ddot{\mathbf{y}}_1 & \cdots & \ddot{\mathbf{y}}_j \\ \vdots & \ddots & \vdots \\ \ddot{\mathbf{y}}_r & \cdots & \ddot{\mathbf{y}}_{r+j} \end{bmatrix},$$

$$\mathbf{Y}_+ = \begin{bmatrix} \ddot{\mathbf{y}}_{r+1} & \cdots & \ddot{\mathbf{y}}_{r+j+1} \\ \vdots & \ddots & \vdots \\ \ddot{\mathbf{y}}_{2r} & \cdots & \ddot{\mathbf{y}}_{2r+j} \end{bmatrix},$$

for j and r are an integer which equals to $2r + j \leq n_k$, the length of sampled response data.

Both ITD and ERA method presented in the following two section are applied to structural identification problem using free-response data or impulse response functions obtained from frequency response functions.

A. Ibrahim Time Domain Method

ITD method in its original formulation has a different approach to construct the block response matrix as stated in eq. (4), the interested reader can refer to [9-12]. The development of the identification program in this research was based on the modified ITD method or known as the modified multi-reference time domain (MMRTD) [19].

A decomposition technique, singular value decomposition or SVD, can be applied to the past of block response matrix $\mathbf{Y}_-$, after building the block response matrix, eq. (4), [19]

$$\mathbf{U}\,\mathbf{S}\,\mathbf{V}^T = svd(\mathbf{Y}_-). \tag{6}$$

S matrix or the matrix of diagonal singular value after SVD possess an information regarding the minimum value of *rank*(q) of the system matrix. It also represents the number degree of freedom of the system or the number of identified structural modes, which is a half of rank.

Finally, the system matrix **A** can be obtained by calculating [19]

$$\mathbf{A} = \left(\mathbf{V}\,\mathbf{Y}_-^T\right)\left(\mathbf{V}\,\mathbf{Y}_+^T\right)^+, \tag{7}$$

where the operator $\square^+$ denotes the Moore-Penrose pseudo-inverse.

B. Eigensystem Realization Algorithm Method

ERA method is a part of system realization which commonly experienced in the control system. Realization process of the system matrix **A** can be obtained by applying the SVD technique as previously stated in eq. (6) [13]. Thus, the system matrix **A** can be gained by simply computed the following quantities [13],

$$\mathbf{A} = \mathbf{S}_q^{-1/2}\,\mathbf{U}_q^T\,\mathbf{Y}_+\,\mathbf{V}_q\,\mathbf{S}_q^{-1/2}. \tag{8}$$

Then, the observation matrix **C** is built from the first block row of $\mathbf{U}_q\mathbf{S}_q^{1/2}$.

C. Modal Parameter Identification

The identification program developed in this research using SPE was oriented for multi-order identification. The number of model order of the identified structure is given more than the minimum rank of system matrix or referred as the overspecified model. Stabilization diagram is applied to distinguish between structural (physical) modes and spurious (noise) modes resulted from the overspecified model. Using this diagram, the identified structural modes tend to be stable at their natural frequencies also the value of damping ratio for each consecutive model order.

Eigenvalue decomposition (EVD) is a de facto technique to extract modal parameters from system matrix **A**

$$\mathbf{\Lambda\Psi} = evd(\mathbf{A}), \tag{12}$$

which yields the structural modes

$$\exp(\boldsymbol{\mu}\tau) = \mathbf{\Lambda};\ \mathbf{\Phi} = \mathbf{C\Psi}, \tag{13}$$

in which $\boldsymbol{\mu}$ and $\mathbf{\Phi}$ denote the diagonal matrix of complex eigenvalues and its conjugates and the matrix of complex eigenvectors and its conjugate. τ is the sampling time. Finally, the identified natural frequencies and damping ratios for the i^{th} mode are given by

$$f_i = \frac{a_i}{2\pi\tau};\ \ \zeta_i = \frac{|b_i|}{\sqrt{a_i^2+b_i^2}}, \tag{14}$$

where $a_i = |tan^{-1}(\Re(\mu_i)/\Im(\mu_i))|$ and $b_i = ln|\mu_i|$.

These natural frequencies and damping ratios are plotted for each model order to determine the structural and spurious modes. This diagram is quite messy because all values extracted using (12) is plotted. An automatic interpretation of this diagram after performing EVD in three stages is proposed by [20] and the fourth stage added by [21] to suppress anomalies in selecting the final physical modes.

The interested readers can refer to both papers for the detail exposition regarding each stage. The automatic interpretation of modal parameter after extraction process used in this research applies those approaches.

D. The Development of Identification Algorithms using SPE

Python programming language has supported scientific computation in parallel ways and can deal with large-scale numerical data. There are lot scientific modules in python, but the main modules for numerical computation are NumPy (numerical python) and SciPy (scientific python). These numerical modules have been linked with Intel MKL (Math Kernel Library), a mathematic library made by Intel to support parallel computation on a machine with multi-core processors. For the ease of use, python and its scientific modules can be installed by using Anaconda python distribution.

The development of the identification program using SPE is referred to the algorithm shown in Fig. 1. This program has the following dependencies:

- Python 3.5 and above,
- NumPy 1.11 + MKL 2017.0,
- SciPy 0.19 + MKL 2017.0,
- Scikit-Learn 0.18 + MKL 2017.0,
- Numexpr 2.6,
- Matplotlib 2.0,

that installed using Anaconda python distribution. The description of each module can be obtained from their official website.

1	**Input:** Response data **y**.
2	**Signal processing:** filters, decimations.

3	**For each model order:**
	Construct the block response matrix $\mathbf{Y}_{-}$, $\mathbf{Y}_{+}$.
	Apply SVD, eq. (6).
	If ITD (MMRTD):
	    Compute **A**, eq. (7).
	Elif ERA:
	    Compute **A**, eq. (8).
	Compute $\mathbf{C} = \mathbf{U}_q\mathbf{S}_q^{1/2}$.

4	**For each model order:** Apply eq. (12), (13), (14).
5	Automatic modal parameter extraction.
6	**Output:** Identified modal parameters (f_i, ζ_i, $\boldsymbol{\phi}_i$) and plot of stabilization diagram.

Figure 1. Algorithm for developing time domain structural identification program.

Both identification methods developed in SPE was put under the python *class* **`fri`**. *Class* fri (free response identification) was a python module designed for the structural identification using free vibration responses, as presented in the third step of in Fig. 1. Meanwhile, the rest of the algorithm was implemented into *class* **`auto_mpe`**, a python module for automated modal parameter extraction.

E. The Validation of Identification Algorithms

Figure 2 depicted the SUT used in this research. It was a cantilever beam with three additional lumped masses clamped on the beam. This SUT was used for evaluating the performance and effectivity of the developed identification algorithms in the SPE. The developed programs would be used to identify the modal parameters of SUT using the free vibration responses at three positions of accelerometers shown in Fig. 2.

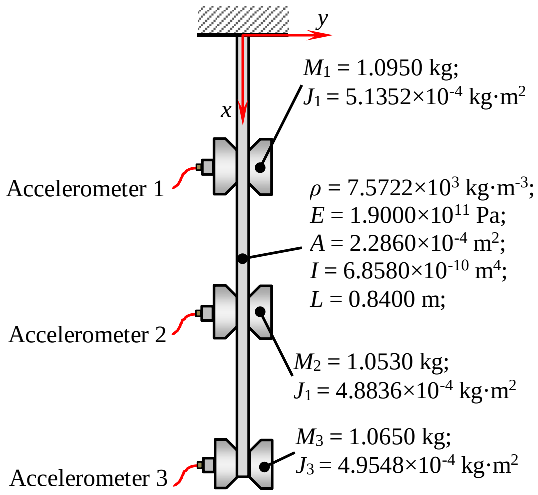

Figure 2. A cantilever beam with three additional lumped masses, the numerical value as presented in [15].

There were two free vibration data of the SUT used in this research, i.e.: numerically simulated responses and captured responses during vibration test. The numerical simulation used the 1D model of the SUT in Fig 2. Both data came from the previous research while conducting during the master research at the Structural Dynamics Laboratory, Mechanical Engineering Department, Andalas University, in 2013-2014 [15]. For this research, the numerically simulated response was added with a Gaussian white noise with SNR of 12.04 dB. This value indicated that the signal strength was four times to the noise strength.

3. Results and Discussion

Both time domain identification program was verified by means of two conditions of free vibration responses. These two conditions were numerically simulated responses and experimentally recorded responses in laboratory scale test. Both conditions were conducted to identify the structural modes below 400 Hz.

A. Numerically Simulated Data

The free vibration responses were generated for the duration of 4 s with the sampled time 0.1 ms. Then, this response was added with white noise with SNR of 12.04 dB. A pre-processing stage was applied to decimate the responses, i.e.: applying low pass filter and down-sampling. The responses were decimated with a down-sampling factor of 4 or 0.4 ms sampled time.

The model order used by both identification method was varied from 2 to 160 with the increment of 2. The system matrix **A** and the observation matrix **C** were recovered for each variation of the given model order. Both matrices yielded from the identification process were extracted to obtain the modal parameters. The extraction process was performed by implementing EVD and four-stage automatic interpretation of modal parameter extraction.

The result from the last effort was plotted into the stabilization diagram for each identification method as shown in Fig. 3. Using this diagram, the physical and spurious modes can be viewed. The physical modes can be found easily from the vertical green circle around their natural frequencies. Meanwhile, the spurious or noise modes were marked by the red cross. All identified physical modes were in 95% confidence interval.

Both identified physical modes from the stabilization diagram (Fig. 3) and previous FEM calculation [15] were presented in Table 1. It can be acknowledged that the discrepancies for the (mean value of) natural frequencies were less than 0.60% between the FEM result and the identified ones. The value of frequencies from the identified ones listed in the Table 1 were the pairs of the mean value and its respective standard deviation. The used of this approach was due to the scattering of identified modes on the stabilization diagram can be treated in a statistical way.

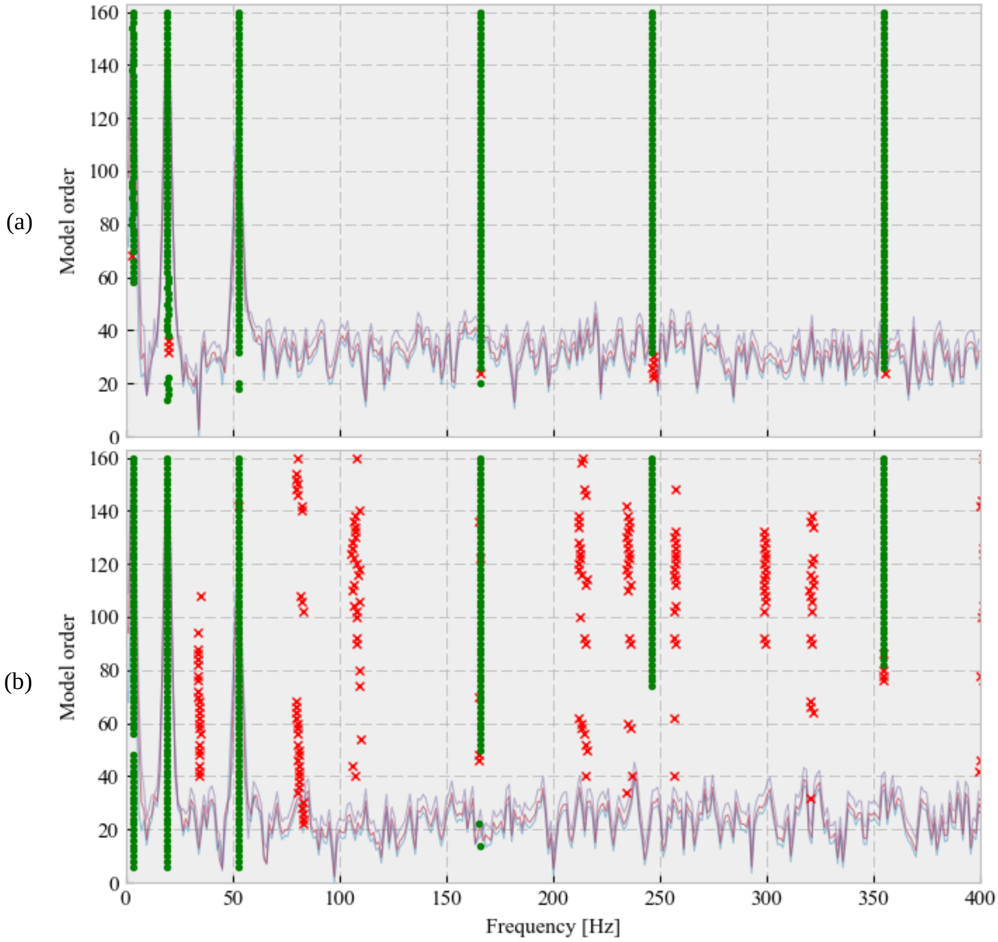

Figure 3. Stabilization diagram of the identified result using (a) ITD (MMRTD) method and (b) ERA method. Green circle [•] and red cross marks [×] denote the symbol for the identified physical and spurious modes respectively.

Table 2 showed the identified damping ratios yielded from both identification methods. The damping ratios used for generating numerically simulated responses were 2.0000% for each mode [15]. Overall, there were below 1% of discrepancies between the identified damping ratios and damping ratios used for generating the responses. Whereas, the first identified modes showed higher discrepancy 3.80% and 6.55% for ITD (MMRTD) and ERA method, respectively.

Table 1. The identified natural frequencies of the SUT in Fig. 2 using numerically simulated responses with SNR of 12.04 dB, f_μ and σ_f denoted the mean and standard deviation of the identified values.

Modes	**FEM** [15]	**ITD (MMRTD)**		**ERA**	
No.	f **(Hz)**	f_μ **(Hz)**	σ_f **(Hz)**	f_μ **(Hz)**	σ_f **(Hz)**
1	3.1037	3.1125	0.0231	3.1041	0.0009
2	19.5281	19.5299	0.0317	19.5281	0.0004
3	52.5920	52.5821	0.0258	52.5873	0.0003
4	164.8033	165.6540	0.0044	165.6433	0.0353
5	246.8868	246.3935	0.0018	246.3921	0.0080
6	354.2033	354.7282	0.0063	354.7300	0.0036

Table 2. The identified damping ratios of the SUT in Fig. 2 using numerically simulated responses with SNR of 12.04 dB, ζ_μ and σ_ζ denoted the mean and standard deviation of the identified values.

Modes	**Simulated** [15]	**ITD (MMRTD)**		**ERA**	
No.	ζ **(%)**	ζ_μ **(%)**	σ_ζ **(%)**	ζ_μ **(%)**	σ_ζ **(%)**
1	2.0000	1.9240	0.3014	2.1310	0.2035
2	2.0000	2.0139	0.1087	1.9998	0.0005
3	2.0000	1.9966	0.0590	2.0019	0.0041
4	2.0000	1.9977	0.0266	1.9881	0.1172
5	2.0000	1.9919	0.0011	1.9914	0.0024
6	2.0000	1.9831	0.0030	1.9837	0.0006

(a) FEM (rows) vs ITD (MMRTD) (columns)

Mode	1	2	3	4	5	6
1	1.00	0.01	0.01	0.18	0.01	0.18
2	0.01	1.00	0.00	0.19	0.02	0.28
3	0.00	0.00	1.00	0.58	0.97	0.50
4	0.41	0.17	0.42	0.94	0.46	0.88
5	0.35	0.25	0.28	0.23	0.46	0.15
6	0.04	0.63	0.30	0.45	0.16	0.54

(b) FEM (rows) vs ERA (columns)

Mode	1	2	3	4	5	6
1	1.00	0.01	0.00	0.74	0.80	0.81
2	0.01	1.00	0.00	0.05	0.11	0.08
3	0.01	0.00	1.00	0.18	0.01	0.07
4	0.40	0.15	0.42	0.86	0.15	0.09
5	0.34	0.26	0.33	0.50	0.55	0.39
6	0.04	0.63	0.25	0.04	0.33	0.37

(c) ITD (MMRTD) (rows) vs ERA (columns)

Mode	1	2	3	4	5	6
1	1.00	0.01	0.00	0.74	0.81	0.81
2	0.01	1.00	0.00	0.06	0.11	0.07
3	0.01	0.00	1.00	0.19	0.02	0.08
4	0.18	0.19	0.59	0.66	0.03	0.00
5	0.01	0.02	0.97	0.26	0.00	0.01
6	0.18	0.28	0.50	0.64	0.04	0.04

Figure 4. Modal assurance criterion (MAC) value in the table form correlated the eigenvectors between (a) FEM and ITD (MMRTD), (b) FEM and ERA, and (c) ITD (MMRTD) and ERA.

Modal assurance criterion (MAC) was applied to determine the correlation between the identified modes and the previous FEM calculation. There were three MAC plots used to represent such correlations, i.e.: FEM – ITD (MMRTD), FEM – ERA, and ITD (MMRTD) – ERA, as depicted in Fig. 4.

The correlation between the identified modes (from ITD (MMRTD) and ERA) and the FEM results showed the best value for, at least, 4 lowest modes. Whereas, the other two led to weak correlation. It might be brought by the noise contained in the responses. This noise destroyed a certain degree of information owned by the responses. Meanwhile, the MAC value between two identified methods only gave 3 lowest modes with the highest correlation, but the rest did not.

B. Experimentally Recorded Data

The free vibration responses of the SUT were generated experimentally by striking the third mass. The free decay responses were recorded immediately for the duration of 1 s with the sampled frequency of 8192 Hz. The performance of the identification results yielded by ITD (MMRTD) and ERA was compared to the result of EMA test with SIMO technique.

The identification process using ITD (MMRTD) and ERA method was conducted with the same procedure as performed for numerically simulated responses. The result gave by time domain identification process was plotted on the stabilization diagram as depicted in Fig. 5 (b) and (c). The identification process was performed by applying a different down-sampling factor to the free vibration responses. Three and ten down-sampling factor were chosen for the identification process using ERA and ITD (MMRTD) method, respectively.

For a comparison, a peak picking method was employed to extract modal information from FRF given by the EMA test, as given in [15]. The FRFs was generated by implementing H_1 estimator from recorded responses and excitation. The notation $\mathbf{H}_{ij}(\omega)$, in Fig. 5 (a), denoted the generated FRFs was computed by measuring the acceleration response at the i^{th} mass while the excitation was applied at the j^{th} mass.

The identified modes, the natural frequencies and damping ratios, shown in Fig. 5 were then presented in Table 3 and 4, respectively. Peak picking and ITD (MMRTD) method only revealed 7 and 9 modes out of 10 modes identified by ERA method. If the identified frequencies from ERA and ITD (MMRTD) method were compared to Peak Picking method which gave the discrepancies below 1%. Thus, time domain methods and peak picking method identified closely the natural frequencies of the SUT from vibration test data.

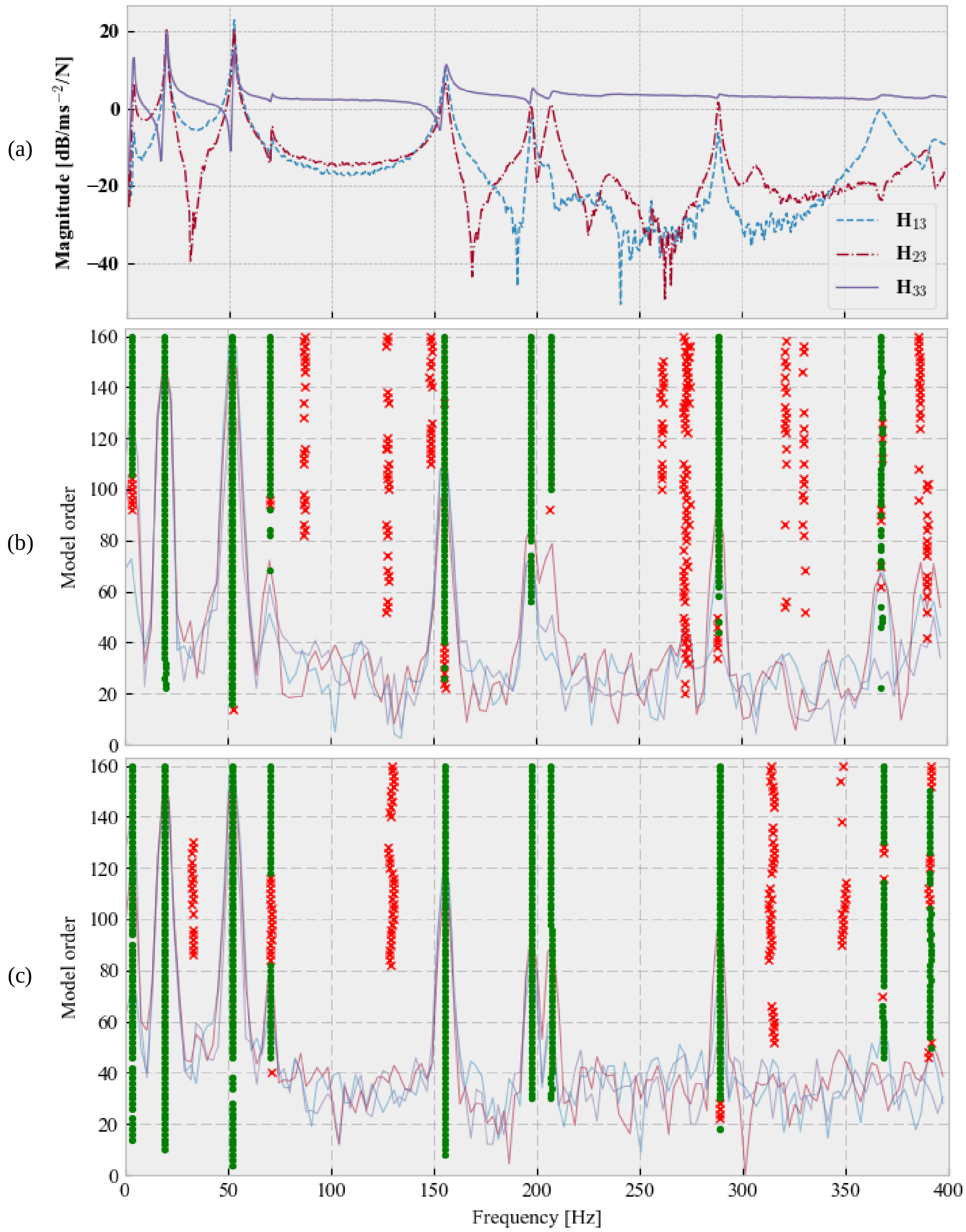

Figure 5. (a) FRFs for peak picking from EMA testing and the stabilization diagram yielded by (b) ITD method and (c) ERA method using time domain responses from free vibration testing. Green circle [•] and red cross marks [×] denote the symbol for the identified physical and spurious modes respectively.

Table 3. The identified natural frequencies of the SUT in Fig. 2 using data from experimental vibration testing, f_μ and σ_f denoted the mean and standard deviation of the identified values, and "–" was the unidentified value.

Modes No.	EMA Peak Picking [15]	ITD (MMRTD)		ERA	
	f (Hz)	f_μ (Hz)	σ_f (Hz)	f_μ (Hz)	σ_f (Hz)
1	3.1875	3.1928	0.0417	3.2176	0.0063
2	19.5000	19.5207	0.0147	19.5055	0.0016
3	52.3750	52.3544	0.0090	52.3570	0.0009
4	70.5625	70.4100	0.0651	70.4053	0.1019
5	155.3750	155.4296	0.0283	155.4666	0.0314
6	197.9375	197.4318	0.0769	197.4744	0.0560
7	–	207.1241	0.0851	206.9367	0.0961
8	288.7500	288.6078	0.0419	288.7634	0.0175
9	–	367.9383	0.1542	368.2143	0.0991
10	–	–	–	390.9108	0.1663

Table 4. The identified damping ratios of the SUT in Fig. 2 using data from experimental vibration testing, ζ_μ and σ_ζ denoted the mean and standard deviation of the identified values, and "–" was the unidentified value.

Modes No.	EMA Peak Picking [15]	ITD (MMRTD)		ERA	
	ζ (%)	ζ_μ (%)	σ_ζ (%)	ζ_μ (%)	σ_ζ (%)
1	2.7000	2.4989	1.7896	1.1832	0.3415
2	0.1400	0.4115	0.3059	0.1415	0.0055
3	0.4100	0.1929	0.1293	0.1313	0.0019
4	3.0200	1.2369	0.9601	1.5999	0.2358
5	6.2900	0.6221	0.5547	0.5043	0.2352
6	3.6400	0.3454	0.0918	0.2855	0.0207
7	–	0.8705	0.1998	0.7763	0.0684
8	3.7600	0.2271	0.3699	0.1688	0.0226
9	–	0.9289	0.5483	0.8255	0.0723
10	–	–	–	0.7606	0.0759

ITD (MMRTD) Mode \ ERA	1	2	3	4	5	6	7	8	9	10
1	0.99	0.06	0.02	0.05	0.47	0.38	0.39	0.25	0.01	0.09
2	0.05	1.00	0.00	0.35	0.22	0.01	0.21	0.46	0.41	0.09
3	0.01	0.01	1.00	0.69	0.32	0.52	0.15	0.05	0.47	0.78
4	0.03	0.38	0.69	0.98	0.04	0.28	0.33	0.32	0.04	0.31
5	0.61	0.16	0.23	0.03	0.93	0.82	0.37	0.17	0.22	0.65
6	0.41	0.05	0.49	0.21	0.93	0.98	0.45	0.18	0.30	0.77
7	0.24	0.45	0.17	0.46	0.12	0.41	0.90	0.91	0.09	0.13
8	0.26	0.54	0.01	0.28	0.02	0.19	0.84	0.98	0.33	0.03
9	0.22	0.55	0.04	0.10	0.01	0.04	0.59	0.82	0.61	0.13

Figure 6. Modal assurance criterion (MAC) value in the table form correlated the eigenvectors between the result given by ITD (MMRTD) method and ERA method.

Different with the identified damping ratios, peak picking method yielded relatively high damping values compared to the time domain counterpart, except for the second mode. The lowest damping values were identified by ERA method. It was only for the first mode and the second mode ITD (MMRTD) and ERA method gave a close value with the result from peak picking method, respectively.

For each model order used, ERA method gave a lower standard deviation of the identified damping ratios than ITD (MMRTD) method did. It showed that ERA method can identified damping ratios with the high stable result at respective identified modes for every increment of model order used.

Then, the correlation between the identified modes – ITD (MMRTD) and ERA – was presented by computing the MAC value between their identified complex eigenvectors as shown in Fig. 6. Meanwhile, the correlation of both time domain method with peak picking method was not calculated. This MAC value indicated that there were 8 modes that correlated closely between the identified modes produced by ITD (MMRTD) method and ERA method. The rest of the identified modes did not correlate at all.

4. Conclusion

The time domain identification program based on response only data for free vibration problem had been presented in this research. Next, the verification of the developed time domain identification program on how their performances were also proved by using numerical and experimental vibration data. It presented lower discrepancies in term of the identification results. It can be known that from the result given in the first stage or identification using the numerically simulated responses, especially in the natural frequencies and the damping ratios. Whilst, the MAC value only ensued four lower modes, to be correlated each other, at least.

Then, the identification process by using experimental vibration test attested lower discrepancies in term of the identified natural frequencies and the MAC value between both time domain methods. These time domain methods evinced the identified damping ratios lower than the counterpart did. In general, the developed time domain identification program had shown its performance and robustness to obtain the modal parameters of the SUT from its free vibration responses. The lack of visualization in displaying mode shape will be added for the future implementation of the identification program.

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